

$$\geq \frac{\ln n}{\ln \ln n} \cdot \frac{1}{d} \quad \text{w.h.p.}$$