

$$d := |B_i| \geq \Omega(\log n)$$