Hierarchical routing for large networks
Performance evaluation and optimization

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Overview

- Hierarchical Routing Schemes
- Minimization of Routing Information
- Path Characteristics
- Static Performance Evaluation
Hierarchical Routing Schemes

- Need for reduction in routing table length.
- Complete info about closer nodes and lesser info about nodes located further away.
- $l$ – New routing table length after clustering.
- $N$ – Routing table length without any hierarchical schemes
- Hierarchical partitioning of networks – m-level Hierarchical Clustering (mHC)
3-Level Hierarchical Clustering
Length of RT at any node is strictly a function of the clustering structure i.e. Number of nodes per cluster, number of clusters per supercluster, etc and the number of levels.

**Fig. 2. A tree representation of a 3-level clustered net.**
Filling the table – Distance vector routing kind.
Initialize all the entries at node s to infinity except for self entries.
On receiving update from t (s and t belong to same kth cluster and not to k-1\textsuperscript{st} cluster) and d(s,t) = 1 then for all common destinations do -

\begin{itemize}
  \item if H(s,dest) > 1 + H(t,dest)
  \item H(s,dest) = 1+H(t,dest)
  \item nexthop(s,dest) = t
\end{itemize}
Question

What is the optimal clustering?

What are the effects on path length due to reduction in table length?
Minimizing Routing Information

- $C_k$ - Kth level cluster defined recursively as a set of $k-1$st level clusters.
- Kth level cluster is identified using a vector of predecessors, $i_{k+1} = (i_m, i_{m-1}, \ldots, i_{k+1})$. Hence $C_k(i_{k+1})$ is the address of a Kth level cluster.
- Degree of kth level cluster is the number of $k-1$st clusters included in $C_k$.
- $N_k(i_{k+1})$ and $N_k$ denote the degree of a particular Kth cluster and all K clusters.
- $N = (n_1, n_2, \ldots, n_m)$ be the degree vector.
• Each node of the network, $C_0(i_1)$, contains an RT with an entry for each $k-1^{st}$ level cluster in the same $k^{th}$ level cluster as $C_0(i_1)$, and this for $k=1,2,\ldots,m$.

$$l[C_0(i_1)] = \sum_{k=1}^{m} n_k(i_m, \ldots, i_{k+1})$$

$$l(m, n) \overset{\Delta}{=} \max \left\{ \sum_{k=1}^{n} \sum_{\text{over all nodes}} n_k(i_m, i_{m-1}, \ldots, i_{k+1}) \right\}$$

• Problem- given : $N$, Minimize $l(m,n)$ subject to

$$N = \sum_{i_m=1}^{n_m} \ldots \sum_{i_k=1}^{n_k(i_m,\ldots,i_{k+1})} \sum_{i_2=1}^{n_2(i_m,\ldots,i_3)} n_1(i_m, \ldots, i_2).$$

(1)
Real-valued solution

- Given $m$

  Prop -1: a) All clusters at all levels are composed of same number of lower level clusters.

  $$n_k(i_{k+1}) = n_k = N^{1/m}, \quad \forall i_{k+1}, k = 1, ..., m;$$

  b) With this optimal assignment, the minimum table length is $T = mN^{1/m}$

  Prop -2: The global optimal clustering is achieved when the number of levels is $m_* = \ln N$ and the degree vector $n$ is such that all components have equal values: $n_k = e = 2.718$ for $k=1...m$. Minimum table length $l_* = e \ln N$
The problem is:

\[
\min : l = \sum_{k=1}^{m} n_k \quad \text{s.t.} \quad \prod_{k=1}^{m} n_k \geq N
\]

Prop 5: There exists a global optimum vector \( n_* \), which is composed of at most two components equal to 2, with all others equal to 3.

\[3^m - x 2^x \geq N \quad \text{where } x \in \{0, 1, 2\}\]

\[
\min \quad l = 3m - x
\]
Prop 6: Given $m$, there exists an optimal vector $n$ which is such that no two components differ by more than 1

$$n_m = \lceil N^{1/m} \rceil,$$

$$n_k = \lceil (N/(\prod_{i=k+1}^m n_i))^{1/k} \rceil \quad k = 2, 3, \ldots, m$$

or any permutation of the above solution.

Optimality with no “Self-entries” in the RT.

Prop 7: There exists a non-degenerate global optimum vector such that $n_k = 2$, $k=1,2,\ldots,m$

$$m_* = \lceil \frac{\ln N}{\ln 2} \rceil$$
Path Characteristics

- Increase in the path length due to RT reduction at the equilibrium.
- Non-Clustered (NCR) VS mHR schemes.
- Assumption 1: All clusters at same level \( k \) are of equal degree \( n_k \), \( k=1,2,\ldots,m \). Also 1-connected cluster subnetwork.
- CER: No routing information describing the internal behavior of a cluster is propagated outside the cluster.
- OBR: The average estimated distance from an exchange node to all the nodes in the cluster will be propagated as the routing info for that cluster.
- $h_{st}^c =$ Length of estimated minimum path from node $s$ to node $t$ as derived from the routing info at node $s$.
- Internal Path
- $h_{st}^i =$ Length of the shortest path from node $s$ to node $t$ included in the lowest level cluster to which both belong.
- Exchange node(e or $e_j$)
- $A_{k \left( i_{k+1} \right)} =$ subset of all exchange nodes which connect cluster $C_{k \left( i_{k+1} \right)}$ with any $K$th level cluster which belongs to same $K+1^{st}$ level cluster.
• $W_{eC_k} = \text{Entry in RT giving internal distance measure for } C_k$.

$$w_{eC_k} = \begin{cases} \frac{1}{|C_k|} \sum_{f \in C_k} h_{ef}^c & \text{for the OBR scheme} \\ 0 & \text{for the CER scheme} \end{cases}$$

$$w_{eC_0} = 0$$

• Prop 8 : Let $s$ and $t$ both belong to same Kth level cluster $C_k$ and not to any lower level cluster. Path length form $s$ to $t$ at equilibrium -

$$h_{st}^c = h_{se_0}^i + h_{e_0t}^c$$

$$h_{se_0}^i + w_{e_0C_{k-1}(t)} = \min_{e_j \in A_{k-1}(t)} \{h_{se_j}^i + w_{ejC_{k-1}(t)}\}$$
Bounds on the increase in path length

- Relative increase in average path length: \( D = \frac{h_c}{h} - 1 \)
- Assumption 2: The diameter of any \( k \)th level cluster subnet is less than or equal to \( d_k \), \( k = 1 \ldots m \).
- Assumption 3: Shortest path between cluster nodes belong to the same cluster.
- Lemma 1: The value \( h_{st}^c \) for any pair of nodes \( s, t \) that belong to same \( k \)th level cluster is
  \[
  h_{st}^c \leq \sum_{j=1}^{k} d_j \\
  \forall s, t \in \text{same } k^{th} \text{ level cluster, } \forall k = 1, 2, \ldots, m
  \]
• Prop 9: Average increase in path length in the network due to the reduction in routing info is

\[ h_c - h \leq \sum_{k=1}^{m-1} \left[ 1 - \frac{n_1 n_2 \ldots n_k - 1}{N - 1} \right] d_k \]

• Lemma 2: Bound on path length on arbitrary nodes \( s, t \)

\[ h_{st}^c - h_{st} \leq \sum_{j=1}^{k-1} d_j \]

\[ \forall s, t \in \text{same } k^{th} \text{ level cluster } C_k, \quad \forall k = 1, 2, \ldots, m \]

\[ h_c - h \leq \sum_{k=1}^{m-1} d_k \]
Performance Evaluation

- Networks considered are all connected graphs to fit an m-level HC.
- Resulting cluster subnets at any level are of diameters bounded by a power law function of the number of nodes in that cluster
  \[ d \leq bn^v + c \]
- If \( N \) is the size of the network, then the average path length
  \[ h = aN^v \]
- \( a = \frac{1}{2}, b = 2, c = -2, v = \frac{1}{2} \)
- \( E \) – bound on relative increase in path length \( D \).
\[ N \to \infty \Rightarrow \begin{cases} h_c/h \to 1, & \text{limit} \ E = 0 \\ l/N \to 0 & \text{as} \ N \to \infty \end{cases} \]

Fig. 8. Bound on the relative increase in path length \( E \), versus the relative table length \( l/N \).
Fig. 9. Lower bound on the ratio of path length without and with clustering versus $l/N$.

Fig. 10. Decrease in table length for a given maximum increase in path length.
Thank you
Questions and Suggestions