Network games

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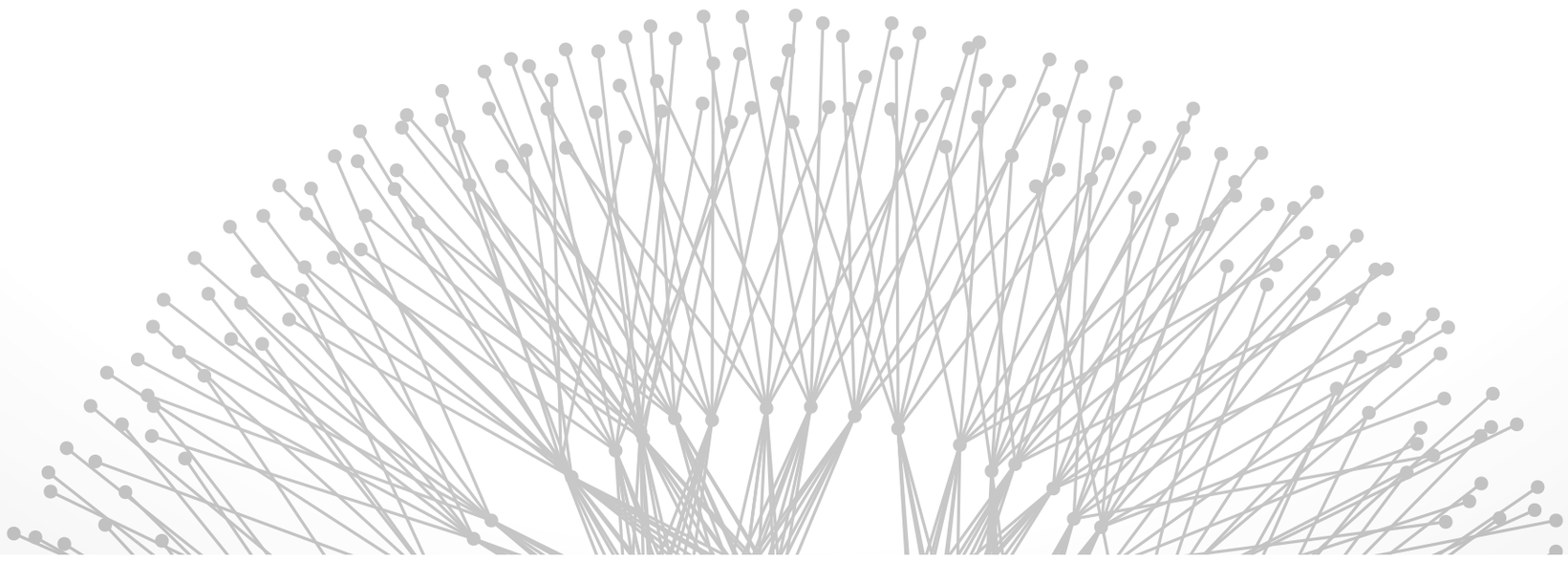
CS 538 September 27 2011



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Demo

Game theory basics

Games & networks: a natural fit

Game theory

Studies strategic interactions between selfish agents

Networking

Enables interaction between remote agents

Networks make games happen!

Game theory

Two or more players

For each player, a set of strategies

For each combination of played strategies, a payoff or utility for each player

		Blue player strategies: Rock	Blue player strategies: Paper	Blue player strategies: Scissors
Red player strategies	Rock	$0, $0	$0, $1	$1, $0
	Paper	$1, $0	$0, $0	$0, $1
	Scissors	$0, $1	$1, $0	$0, $0

(Pure) Nash equilibrium

A chosen strategy for each player such that no player can improve its utility by changing its strategy

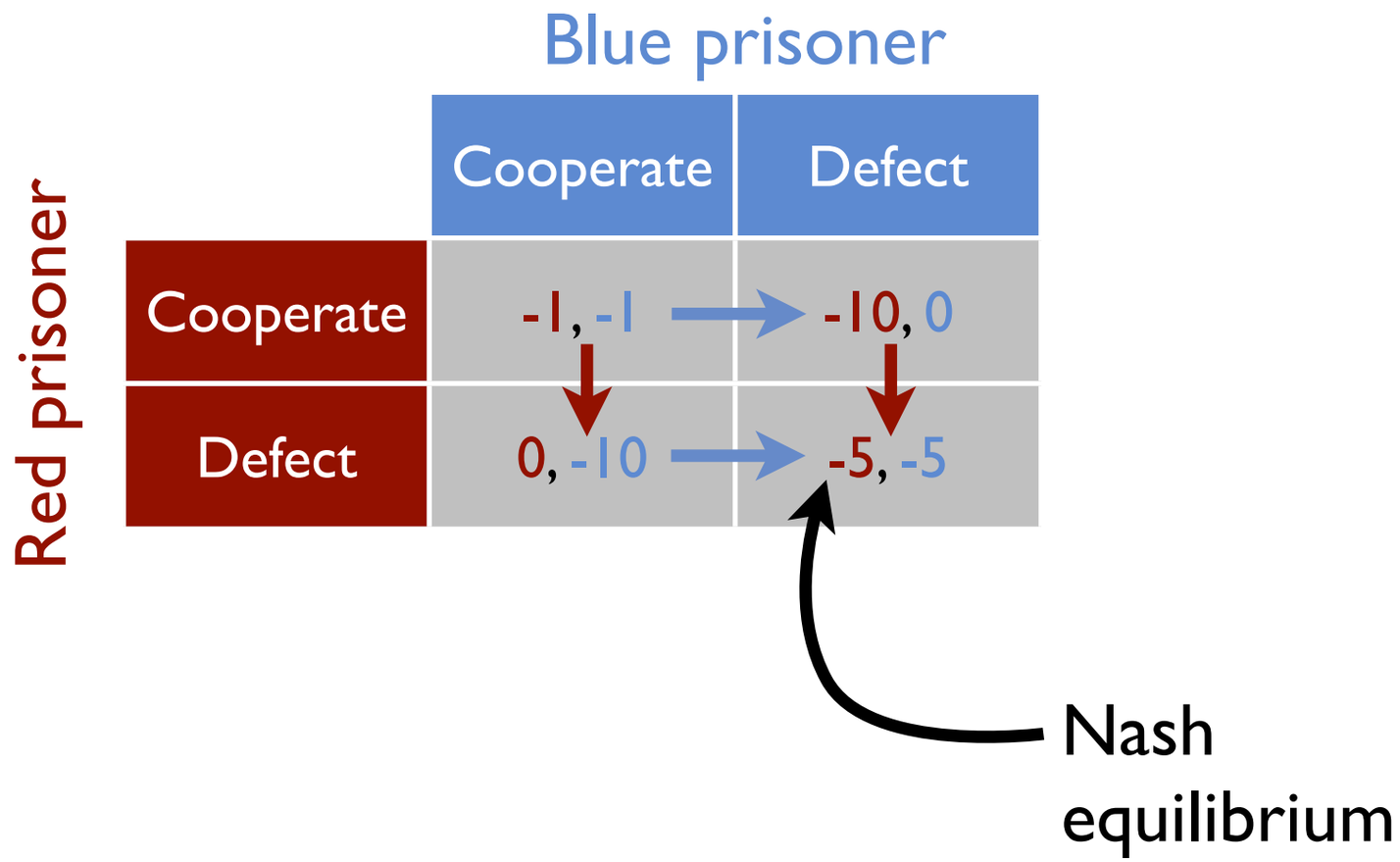
- (In mixed Nash equilibrium: players randomize their strategies according to some distribution and no player can improve its *expected* utility)

Find a Nash equilibrium in Rock-Paper-Scissors:

Red player strategies \ Blue player strategies	Rock	Paper	Scissors
Rock	$0, $0	$0, $1	$1, $0
Paper	$1, $0	$0, $0	$0, $1
Scissors	$0, $1	$1, $0	$0, $0

No pure Nash equilibrium!

Prisoner's dilemma



Price of Anarchy

[C. Papadimitriou, "Algorithms, games and the Internet", STOC 2001]

Assumes some global "cost" objective, e.g., social utility (sum of players' payoffs).

$$\text{Price of anarchy} = \frac{\text{worst Nash equilibria's cost}}{\text{optimal cost}}$$

Red prisoner \ Blue prisoner	Cooperate	Defect
Cooperate	-1, -1	-10, 0
Defect	0, -10	-5, -5

Here, PoA = 10/2 = 5.

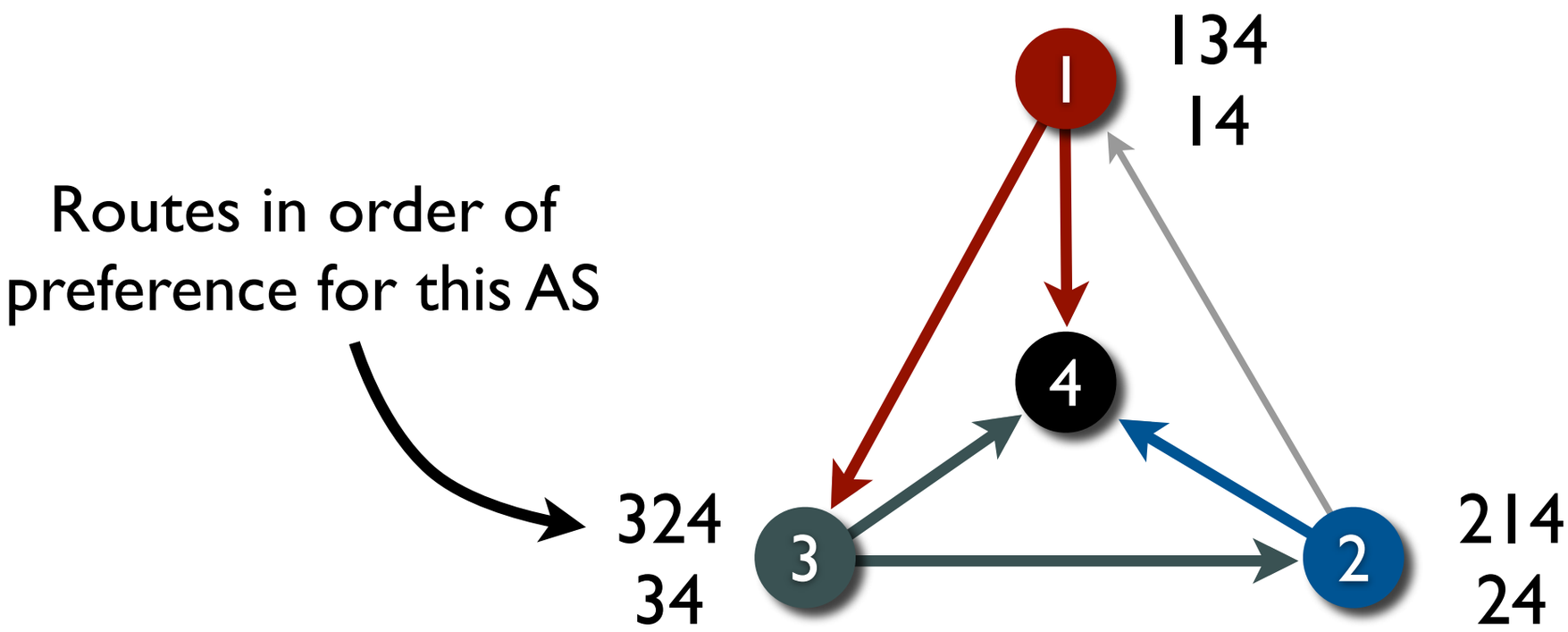
Today's papers

Stable Paths Problem (BGP convergence)	In a game model of BGP, in some instances no Nash equilibrium exists. NP-complete to decide whether one exists.
How bad is selfish routing?	For the selfish routing game, the Nash equilibria are fairly close to optimal under certain conditions (i.e., price of anarchy is small).
Selfish routing in Internet-like environments	In practice the Nash equilibria are very close to optimal latency.

Internet routing as a game

BGP routing as a game

players	autonomous systems
strategies	pick a route, any route... (to fixed dest.)
player's utility	arbitrary function of route (but $-\infty$ for 'illegal' route not offered by neighbor)



No equilibrium!

BGP routing as a game

In general, NP-complete to decide whether an equilibrium exists

Might have 0, 1, 2, 3, ... equilibria

Even if it has an equilibrium, might not converge to it

- Depends on starting state, message timing, ...
- PSPACE-complete to decide whether a given set of BGP preferences can oscillate [Fabrikant, Papadimitriou, SODA'08]

If we assume customer-provider-peer and valley-free routing, guaranteed to converge [Gao, Rexford]

How bad is selfish routing?

The selfish routing game

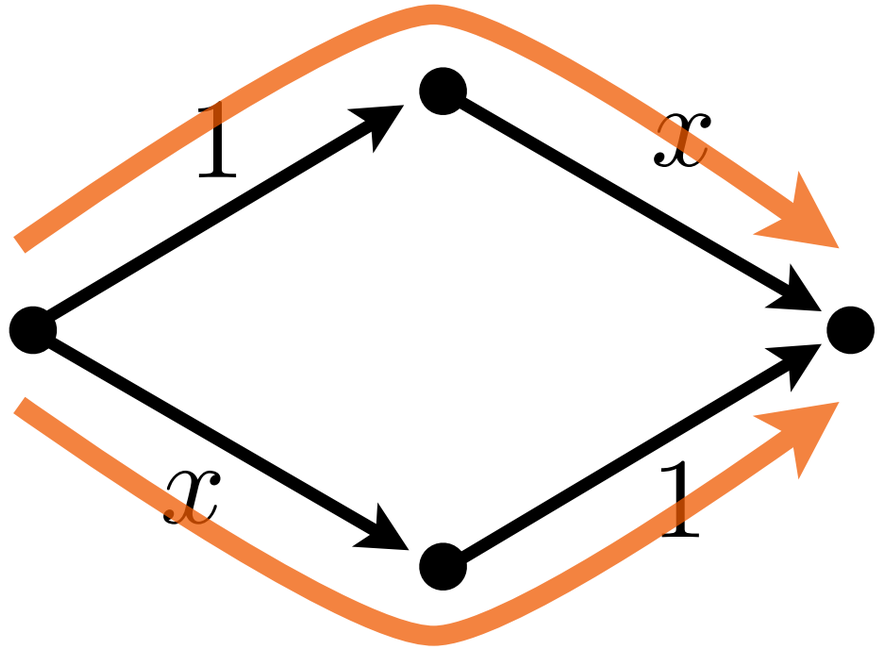
Given graph, latency function on each edge specifying latency as function of total flow *x* on edge

Path latency = sum of edge latencies

Player strategy: pick a path on which to route

Players selfishly pick paths with lowest latency (source-controlled routing)

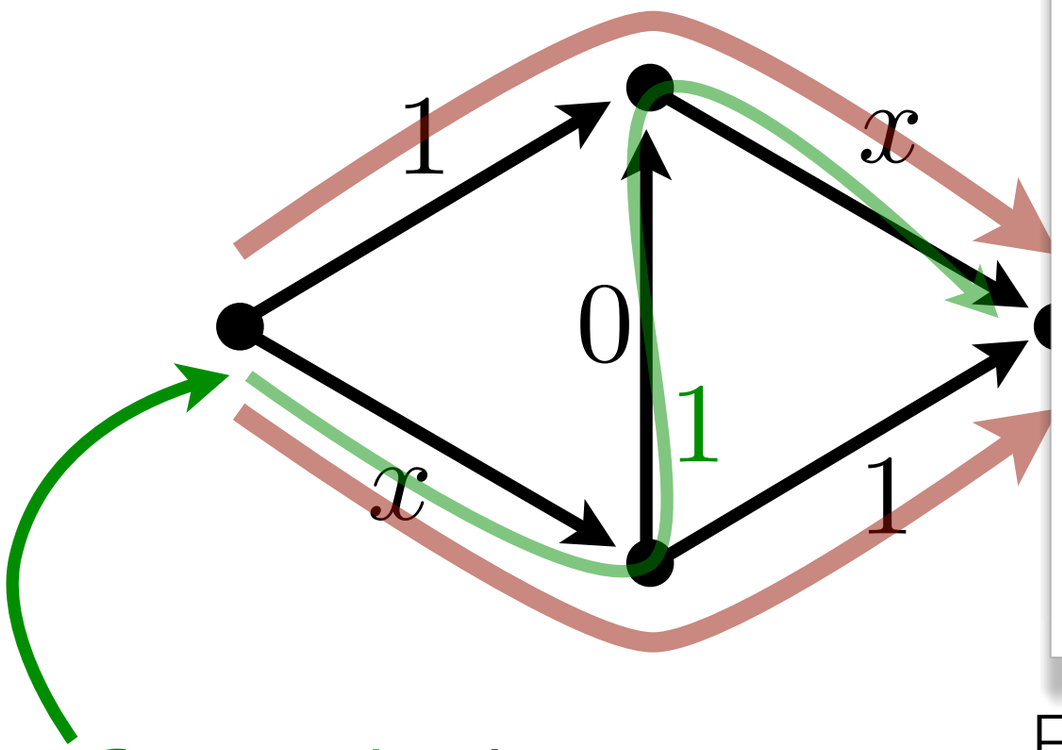
For now assume many users, each with negligible load; total 1



Flow *x* = 0.5 on each path;
Total latency = 1.5

Example: Braess's paradox

[Dietrich Braess, 1968]



Green path is better.
Everyone switches to it!

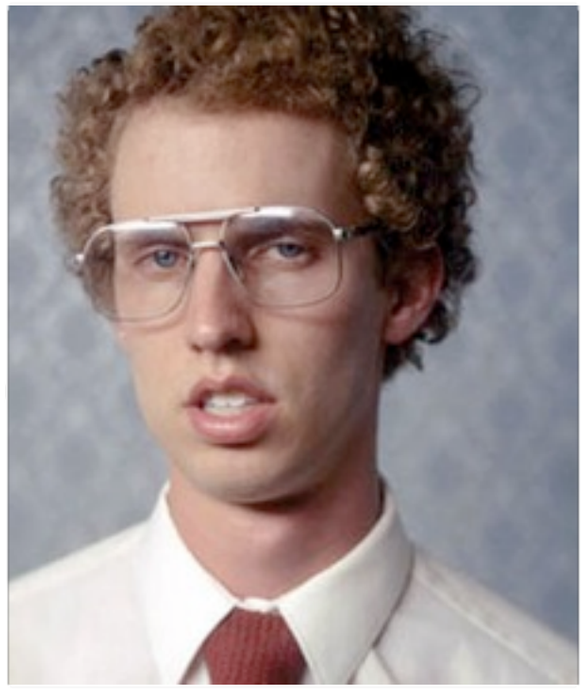


Fig 1b: N. Dynamite.

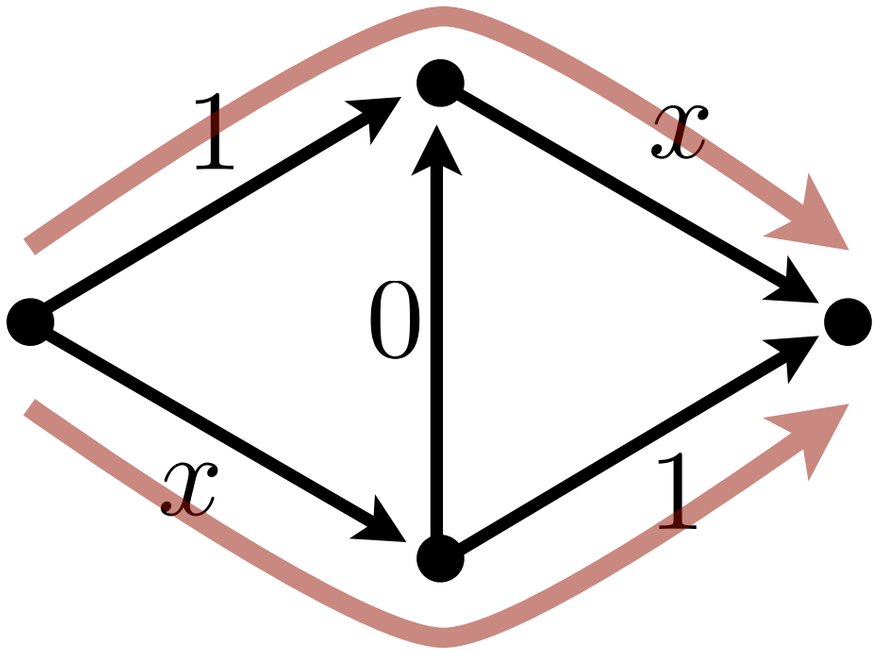
≈

Fig 1a: D. Braess.

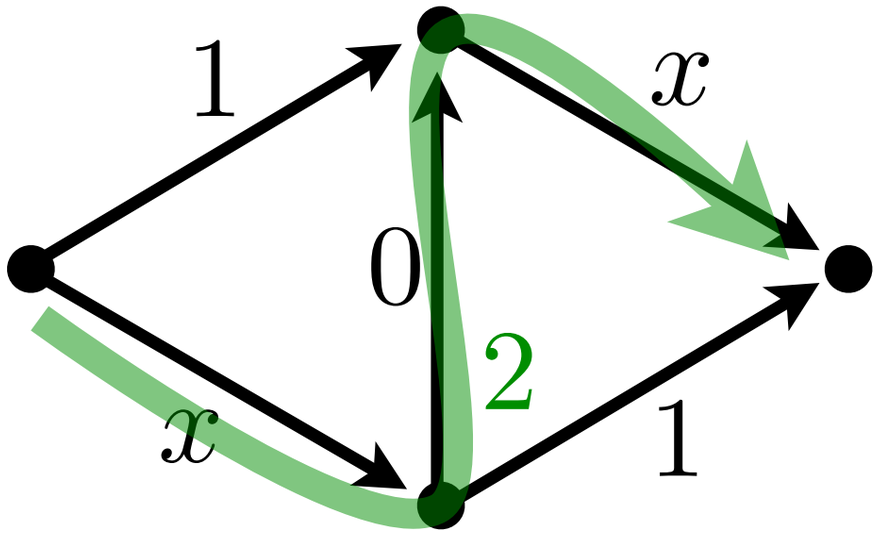
Initially: 0.5 flow along each path; latency 1+0.5 = 1.5

With new edge: all flow along greed path; latency = 2

Example: Braess's paradox



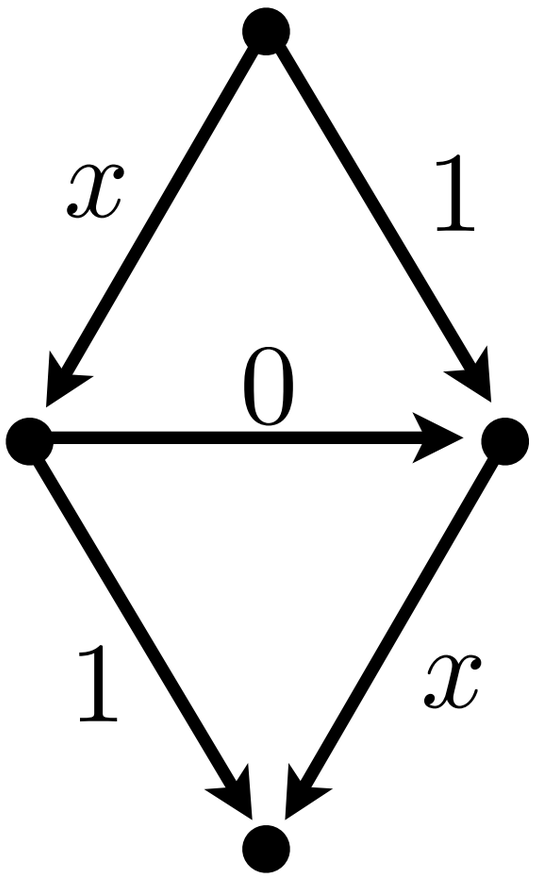
Optimal latency = 1.5

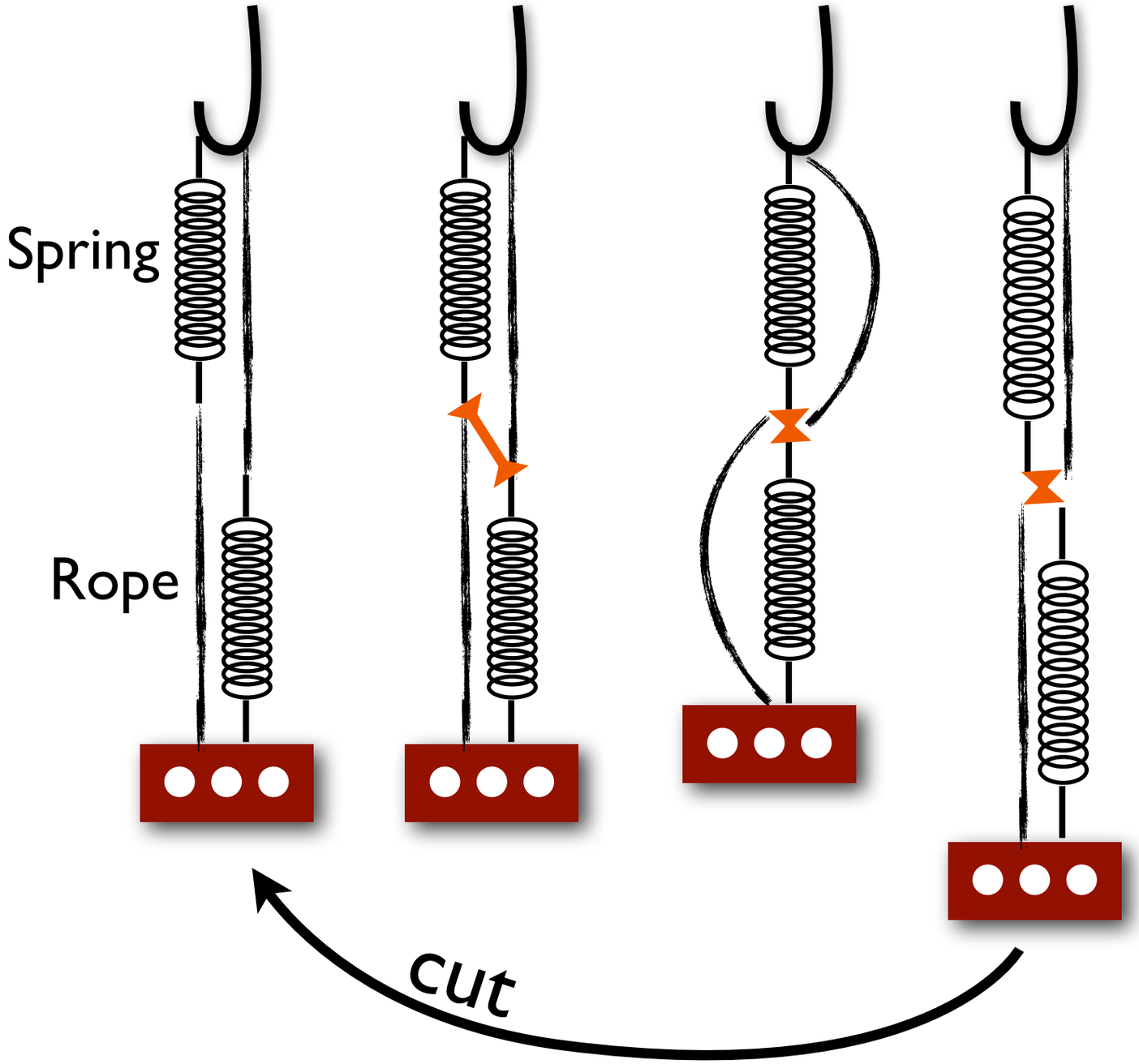


Nash equilibrium latency = 2

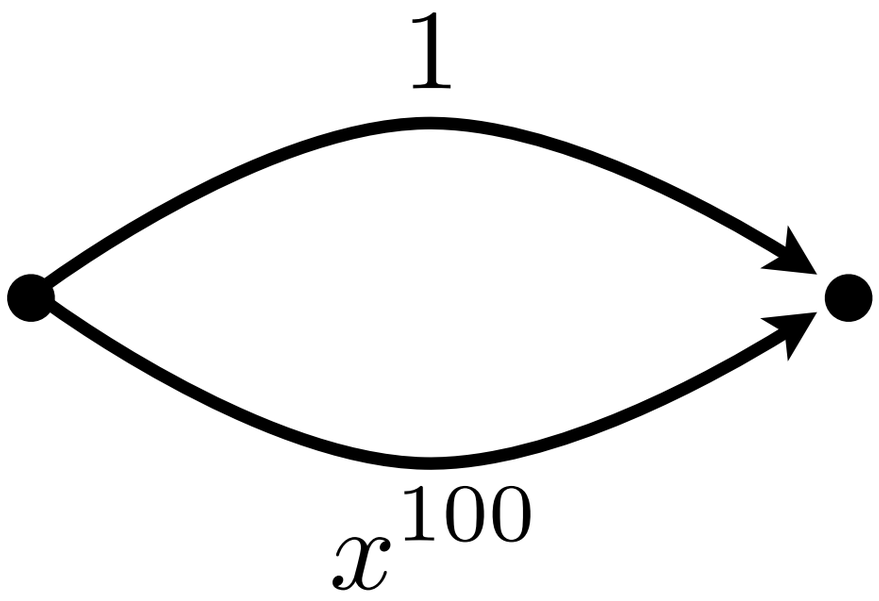
Thus, price of anarchy = 4/3

From links to springs

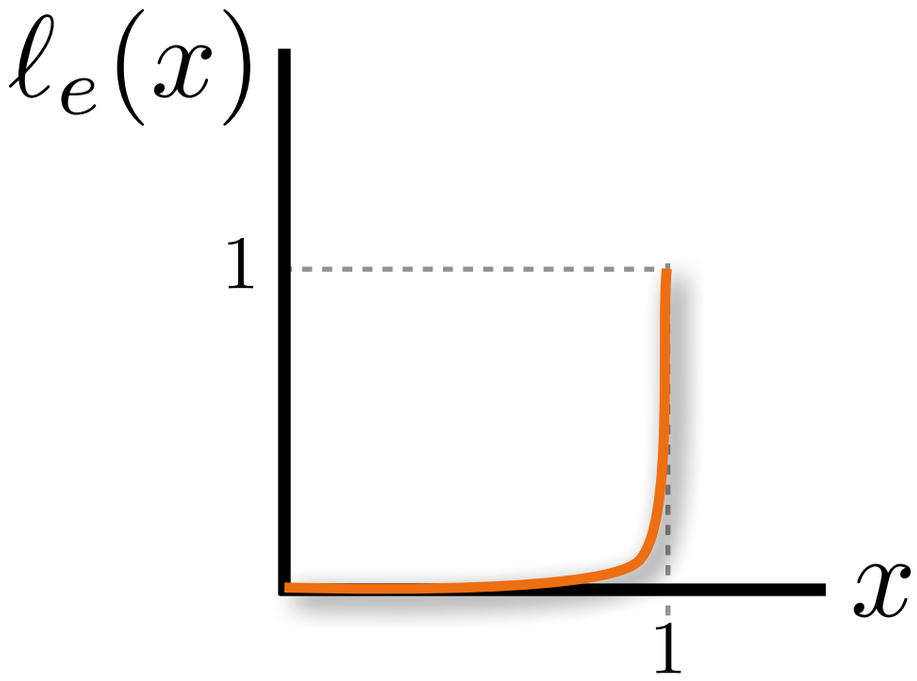




Example: arbitrarily bad



Optimal: almost all flow on bottom; total latency near zero



Nash: all flow on bottom; total latency = 1

Roughgarden et al. results

As we just saw, price of anarchy can be arbitrarily high

But for linear latency functions: PoA $\leq$ 4/3

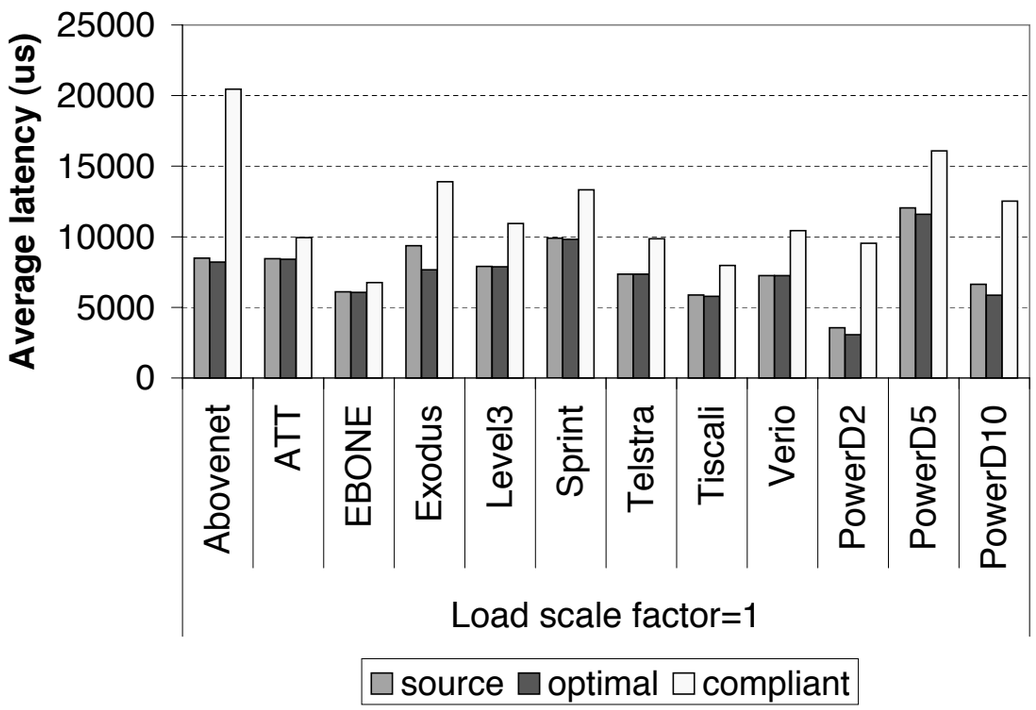
For any latency function: Nash cost is at most optimal cost of 2x as much flow

Extension to finitely many agents

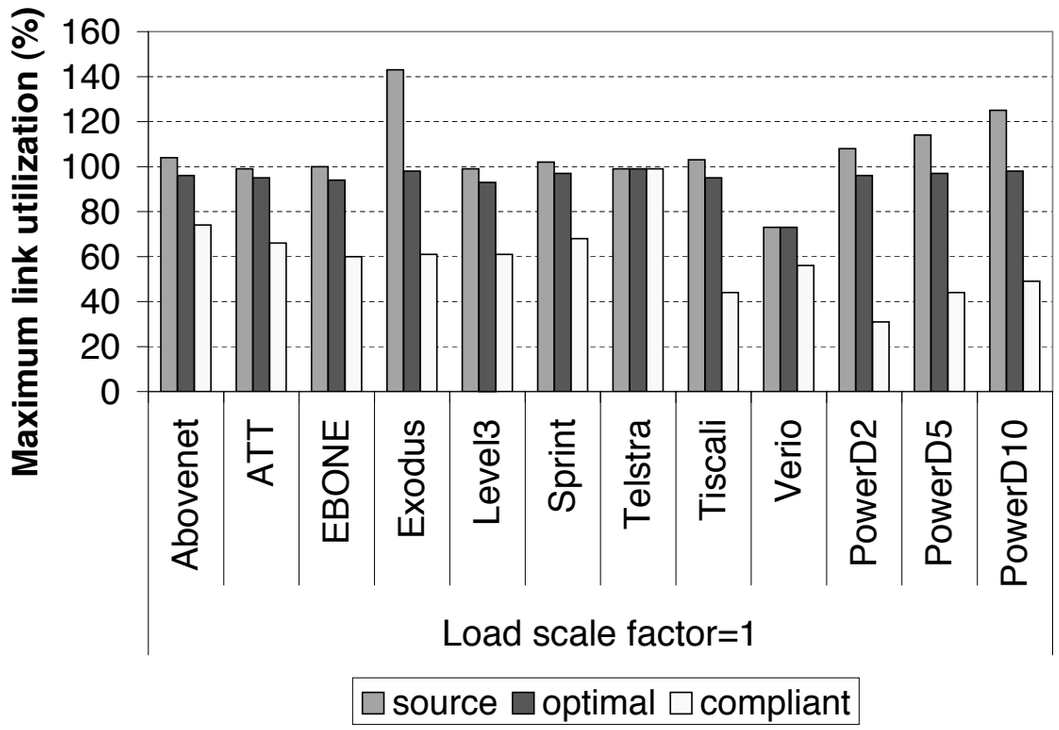
- i.e., a single agent might have a nontrivial fraction of the total bandwidth
- Splittable flow: similar "2x" result
- Unsplittable flow: can be very bad

Selfish routing in realistic networks

[Qiu et al., SIGCOMM 2003]



Close to optimal latency



...but higher maximum link utilization

Conclusion

Selfishness can produce suboptimal routes

- but not too bad, in some cases

Is this a realistic model?

Why are the results important or useful?

Many more applications of game theory to CS (and CS to game theory).

- See Nisan, Roughgarden, Tardos, Vazirani's book Algorithmic Game Theory, available free online!

Announcements

Thursday

- Survey of BGP security Issues (Butler, 2010)
- Hongyang presents on secure ad-hoc routing

Tuesday Oct 4: Michael Schapira, Hebrew U & Google

- 3:00 p.m. - 4:00 p.m., SC 4403
- Talk: (How) Can We Secure Interdomain Routing?